RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

SECOND YEAR [2018-21]

B.A./B.Sc. THIRD SEMESTER (July – December) 2019

Mid-Semester Examination, September 2019

Date : 18/09/2019 **MATH FOR ECONOMICS (General)**

Time : 11 am – 12 noon **Paper : III** Full Marks : 25

[Use a separate Answer Book for each group]

Group – A

Answer **any three** from **Question Nos. 1 to 5** : [3×5]

1. For the function $f(x,y)=\begin{cases}(x^2+y^2)\log(x^2+y^2), \text{for}(x,y)\neq(0,0)\\ 0 \qquad\qquad\quad , \text{for}(x,y)=(0,0)\end{cases}$

   a) State whether the condition of Schwarz's theorem is satisfied or not .

   b) State whether $f_{xy}(0,0)=f_{yx}(0,0)$.

   c) What conclusion do you derive from (a) and (b) ? [2+2+1]

2. a) Determine whether the set $S=\{(x,y): x \text{ and } y \text{ are both rational}\}$ is open or not (justify) . Also find the boundary of the set. [2+1]

   b) Find the directional derivative of $f(x,y)=xe^{-yx}+xy$ at $(1,1)$ in the direction of unit vector $\frac{1}{5}(3,4)$. [2]

3. a) Are the functions u= x+y-z , ν = x-y+z, w= $x^2+y^2+z^2-2yz$ are functionally related ? [2]

   b) Investigate the extreme values of $f(x,y)=2(x-y)^2-x^4-y^4$. [3]

4. Let, $f(x,y)=\begin{cases}x^2\sin\frac{1}{x}+y^2\sin\frac{1}{y} & When\ x\neq 0, y\neq 0\\ x^2\sin\frac{1}{x} & When\ x\neq 0, y=0\\ y^2\sin\frac{1}{y} & When\ x=0, y\neq 0\\ 0 & When\ x=0, y=0\end{cases}$

   a) Does $f_x(x,y)$ *and* $f_y(x,y)$ are continuous at (0,0) ?

   b) What do you say the differentiability of f(x,y) at (0,0) .

   c) In what way continuity of f_x and f_y related to differentiability. [2+2+1]

5. a) If $u=f(x^2+2yz, y^2+2zx)$, then prove that $(y^2-zx)\frac{\partial u}{\partial x}+(x^2-yz)\frac{\partial u}{\partial y}+(z^2-xy)\frac{\partial u}{\partial z}=0$. [3]

   b) If $v=f(u)$, where u is a homogeneous function of x and y of degree n , then prove that $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=nu\frac{dv}{du}$. [2]

Group – B

Answer **any two** from **Question Nos. 6 to 9**: [2×5]

6. a) Determine the order and degree of the differential equation $\frac{d^2y}{dx^2}=5\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{2}{3}}$ [1+1]

   b) Determine the differential equation whose primitive is $y^3=2kx+k^2$, where k is a constant. [3]

7. a) Solve: $\frac{dy}{dx}=\frac{x+y+1}{3x+3y+1}$ [5]

8. Examine whether the equation $xy\,dx+\left(2x^2+3y^2-20\right)dy=0$ is exact or not . Hence solve the differential equation . [5]

9. Solve: $\frac{dy}{dx}=\sin 2\frac{y}{x}+\frac{y}{x}$ subject to the condition $y=\frac{\pi}{4}$ at $x=1$. [5]

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